*J. Fluid Mech.* (1981), *vol.* 110, *pp.* 497–502
*Printed in Great Britain*



# REVIEWS

**Geophysical Fluid Dynamics.** By JOSEPH PEDLOSKY. Springer, 1979. 624 pp. DM 79.50.

The fluid mechanics of the natural world is a subject of remarkable span. From early theories of a few phenomena close at hand, such as water waves and flow around obstacles, it has followed the tracks of explorers throughout the atmosphere, oceans and beyond them to the solid earth and the planets. Fridtjof Nansen's studies of ocean dynamics on the *Fram* (amidst polar exploration), the films of Jovian turbulence and reconstructions of mantle convection all give us cause to celebrate the strength and generality of the science.

Pedlosky's text describes an important part of this subject, the geostrophic motions of atmospheres and oceans in which planetary rotation (at a rate $\Omega$) provides a dominant force. These occur typically at length scales greater than a few kilometres, and time scales greater than a day. The book is a long and carefully designed work, expressing the fundamentals laid down in many research papers, and is one that is likely to act as a standard for post-graduate courses. Such courses would logically follow a basic course of fluid dynamics while running in parallel with more descriptive treatments of oceans and atmospheres and more specialized courses in wave propagation, instability and turbulence.

The approach of the book is to treat in great detail a small set of topics, particularly geostrophic balance, potential vorticity, Rossby waves, the circulation of a homogeneous ocean, and baroclinic instability. There are shorter sections on long gravity waves, frontogenesis, equatorial waves, and steady circulation of stratified fluids.

The basic derivations are exceptionally clear. For these alone, the book would be a valuable authority. There is a long-range design by which the important role of planetary sphericity in treating waves and circulation is introduced gradually: first through the sloping bottom analog, then Rossby's heuristic $\beta$-plane, and finally by treatment of the full spherical geometry. In the latter case the derivation is a formal expansion in powers of the Rossby number, with a description of how the remaining parameters (fluid depth/planetary radius, Coriolis frequency/buoyancy frequency, horizontal scale/fluid depth) select among phenomena. There is no mention of effects like equatorial shear layers parallel to the rotation axis, which dominate outside the geophysical parameter range. Indeed, there is a subtle contest throughout rotating spherical shells between rotational 'stiffness', which tries to keep fluid lines once parallel to $\Omega$ always parallel, and the thinness of the shell and the density stratification, which tend to pick out only the local vertical component of $\Omega$.

The paradigms here are linearized Rossby waves, linearized homogeneous-fluid circulation, and linearized instability theory. Classical wave theory, with basic ideas of group velocity, leads systematically to more complex modes involving density stratification, bottom topography, mean flow, and equatorial effects. Students used to 'good' waves (sound, light, water waves) which may propagate many wavelengths without serious mutual interaction, self-distortion, or dissipation may be lulled into forgetting that Rossby waves, by contrast, often have steepness close to unity.

Nonlinearity is in fact described in the areas of inertial boundary currents, wave-triad interactions and baroclinic instability. More modern topics involving strong nonlinearities are introduced with tantalizing brevity. Numerical simulation models, geostrophic turbulence theory, solitons and the nonlinearity of low-order spectral truncations pioneered by Lorenz all could, with slightly more attention, have added perspective to the linear theory.

Spin-up (the process by which rigid boundary conditions affect the interior through viscous layers and induced secondary flow) is dealt with succinctly, but without mention of the unsolved aspects of the problem for stratified rotating fluids.

Wave–mean-flow interaction, a topic where considerable recent progress is evident, is described in a 'static' model which avoids discussion of the true strength of the effect. A 'non-interaction' theorem is proved which shows that in the absence of dissipation a steady wave field yields no average stress on the Eulerian-mean circulation. This raises such an emotional response in many authors that the alternative term 'non-acceleration' is rising in popularity. For these same waves altered the mean flow when they first arrived and would alter it back again should they depart. It just happens that in a non-dissipating, steady state they have done their work.

The discussion of steady wind-driven circulation of an homogeneous ocean is lengthy, that of the stratified ocean is very brief. This reflects the uneven progress in the field. It also shows both the strength and weakness of the approach of rigorous modelling: 'The homogeneous model of the ocean circulation is in many ways archetypical of geophysical fluid dynamics. The model is physically extremely crude and, *a priori*, makes no attempt to include the vertical structure of the circulation it predicts . . . Nevertheless it is striking how successful the homogeneous models are in offering a compelling simple explanation of the overall pattern of ocean circulation and in particular its western intensification.'

The resemblance between the homogeneous and stratified flows is not coincidental, for Sverdrup's linear relation between wind-stress curl and north-south vertically integrated flow holds in either case (provided the sea floor is flat). The widely used word 'relation' rather than 'equation' or 'approximation' is, incidentally, appropriate. Rather in the spirit of a constitutive relation it expresses the conservation of projected length (upon $\Omega$) of line segments moving with the fluid. In homogeneous or density-layered fluid rapid rotation endows the fluid with 'stiffness' along lines parallel to $\Omega$. Sverdrup's relation simply describes how fluid in this spherical annulus moves with north-south velocity just sufficient to avoid compression or extension of these line segments, given the wind-induced vertical flow diverging from the surface boundary layer. Concentrating on the vertically averaged flow (and calling it 'the circulation') does, however, hide the essential three-dimensionality. Beneath the great gyres of wind circulation are complex arteries of density-driven flow, for example the jet-like current along the western boundary of the North Atlantic beneath the Gulf Stream, running opposite to it, which carries water from the north to ventilate the deep oceans throughout the world. The emphasis on the vertically averaged flow could be balanced by discussing this three-dimensionality, if only through the simple models of Stommel and Arons.

There is a challenge in writing a book like this to include some of the special insights that are scattered about the research literature: those rare remarks that crystallize our ideas about an entire range of flow phenomena. Pedlosky succeeds well, for example, in describing the western intensification of both eddies and circulation as a consequence of Rossby wave reflection properties, in using Taylor's definition of instability (as being growth in the squared displacement of fluid particles rather than growth of a single normal mode) to derive the necessary condition for instability, and in recounting Moore's ocean circulation theory, in which the required western-boundary dissipation occurs not in the boundary layer but in a standing Rossby wave train executed by the flow as it leaves that boundary (this and the related computer simulations of Bryan provided correct intuition for the later 'eddy-resolving' models). Each of us could probably have suggested additions: my personal list would include the use (due to Lighthill) of wave-group velocity in the limit of vanishing frequency to describe the set-up of Taylor columns and westward influence on a $\beta$-plane; the way (described above) in which planetary rotation endows a homogeneous fluid with stiffness parallel to $\Omega$; the eddy transport of potential-vorticity (often the product of Taylor's diffusivity and the mean potential vorticity gradient) as a force on the Eulerian mean fluid, and, generally speaking, the role of the geometry of the mean potential vorticity field in setting the stage (and pathways) for wave propagation, instability, steady free circulation, and eddy induced circulation.

*Geophysical Fluid Dynamics* is clear, well organized and sharply focused. The choice of depth over breadth makes this a pedagogically effective and relatively self-contained text rather than a survey, monograph, or handbook. It will be a lasting source for both the formulation and, by example, the execution. As does any good teacher, it speaks with patience and authority. A young student, just learning English, perhaps summed up the prevailing feeling. On his application to graduate school in Woods Hole he entered, under 'outside interests', 'J. S. Bach, Kierkegaard and Joseph Pedlosky'.

P. RHINES

**An Introduction to the Mathematical Theory of Geophysical Fluid Dynamics.** By SUSAN FRIEDLANDER. North-Holland, 1980. 272 pp. £16.25.

The recent books *Stratified Flows* by Yih and *Buoyancy Effects in Fluids* by Turner on motion of a fluid of variable density, *The Theory of Rotating Fluids* by Greenspan and *Geophysical Fluid Dynamics* by Pedlosky on motion of a fluid in a rotating frame, and *Magnetic Field Generation in Electrically Conducting Fluids* by Moffatt, leave a gap – a book on geophysical fluid dynamics as a whole. Friedlander takes a distinctive view, but leaves the gap unfilled. In spite of her title, she follows Greenspan and Pedlosky in concentrating on the motion of a fluid in a rapidly rotating frame, i.e. at small values of the Rossby number, and considering stratification only in combination with rotation. It is a pity if geophysical fluid dynamics is to be defined as excluding mesoscale motion of the atmosphere and oceans and motion of the interior of the earth. However, the present book is a textbook designed for applied mathematics graduate students who are interested in the theory of motion of a fluid in a rotating frame rather than in particular applications to meteorology and oceanography. The book is short and to the point, with a few problems at the end of each chapter; these are its distinctive merits.

The publishers describe the book as one of a series of mathematical notes. This is an apt description because the pages are printed reproductions of a 'camera-ready' typescript. Not only does the text look like lecture notes, but it also contains those misprints, minor errors and infelicitous or imprecise phrases which one associates with notes rather than a book. Most of this roughness is not of great significance, but some is important; for example, a student reading page 112 might think that boundary currents usually occur on the eastern side of an ocean basin in the southern hemisphere as well as the western side in the northern hemisphere; again, the definition of the Brunt–Väisälä frequency on page 127 involves the angular velocity of the frame of reference.

The titles of the chapters are Equations of Motion, Potential Vorticity, Non-dimensional Parameters, Geostrophic Flow, The Ekman Layer, The Geostrophic Modes, Inertial Modes, Rossby Waves, Vertical Shear Layers, Analogies Between Rotation and Stratification, The Normal Mode Problem for Rotating Stratified Flow, Rossby Waves in a Rotating Stratified Fluid, Internal Waves in a Rotating Stratified Fluid, Boundary Layers in a Rotating Stratified Fluid, Spin-down in a Rotating Stratified Fluid, and Baroclinic Instability. Most of the standard material is there but, as authors do, Friedlander emphasizes her personal interests. In so short a book she devotes 58 pages (Chapters 11–13) to the specialized topic of small oscillations of a stratified fluid confined in a rotating container. This is given without any previous treatment of the similar and more fundamental problem when the container does not rotate, and with no treatment of topics such as barotropic instability and the quasi-geostrophic approximation, which many might judge to be more important for students to learn. Again, surely it would have been better to have introduced elementary Kelvin waves and internal gravity waves for their own sakes, rather than as special cases of more complicated problems. A textbook on geophysical fluid dynamics as a whole could have more naturally developed these topics in a way suitable for a student.

P. G. DRAZIN

**Fluid Mechanics, A Concise Introduction to the Theory.** By CHIA-SHUN YIH. West River Press, 1977. 622 pp. $19.80.

This is a corrected edition of the book first published by McGraw-Hill in 1969. It is still a hardback, although the colour and design of the cover have been changed. It is on the whole well produced, although the intensity of the new printing is uneven in places. I have found some minor corrections of, and additions to, the text, but no change in contents or pagination. Indeed, hardly anything of importance seems to have been changed, not even the price. The book cost $20.50 in 1969 and the present edition costs $19.80! I mention all these details because Professor Yih has shown enterprise as publisher, as well as author, in following the admirable example of Professor Van Dyke's publication of the second edition of his *Perturbation Methods in Fluid Mechanics*. It is, of course, much cheaper to publish the second impression of a book than the first, because the editing and type-setting need be done only once. It may be hoped, nonetheless, that the larger scientific publishers will follow West River Press in cutting their overheads lest the rise in the price of books on fluid mechanics leaves libraries as the only buyers.

'This book is primarily intended for first- or second-year graduate students in American universities . . .' writes the author. He notes that no previous acquaintance with fluid mechanics is assumed, although I feel that most readers without that acquaintance will find the book too demanding. Of course, thorough teaching may make the book assimilable, but the topics covered are diverse and, in many places, deep or specialized.

The coverage of the book is indicated by the titles of the ten chapters: Fundamentals, The Basic Equations, General Theorems for the Flow of an Inviscid Fluid, Irrotational Flows of an Inviscid Fluid of Constant Density, Waves in an Incompressible Fluid, The Dynamics of Inviscid Compressible Fluids, Effects of Viscosity, Heat Transfer and Boundary Layers of a Gas, Hydrodynamic Stability, and Turbulence. There are two appendices, and several problems at the end of each chapter. Readers will inevitably disagree concerning the optimum coverage in an introductory text on fluid mechanics, but I feel myself that, in general, the coverage here is comprehensive and the treatment sound. Most important topics are included, though lubrication theory seems too important, especially for engineers, to have been omitted. Waves in fluids and geophysical fluid mechanics are treated extensively.

However, one chapter is notably below the standard of the rest of the text. The last chapter does not illuminate the major physical properties of turbulence and their relative importance. Turbulent jets and wakes are not treated (although the spread of a turbulent jet is the subject of one short problem). The section on turbulent flow in pipes does not give explicitly the law of resistance. There is no mention of a logarithmic mean velocity profile or of Kármán's constant. Kolmogoroff's and Obukhoff's dimensional analysis of turbulence is deservedly presented, and isotropic turbulence is analysed at length, but the 5/3 decay law of the energy spectrum is excluded.

The order of topics conforms by and large to my taste, although the discussion of boundary layers before flow at small Reynolds numbers does not. Also the duplicate treatments of characteristics on pages 211–219 and 288–294, and of Howard's semicircle theorem on pages 472–473 and 500–501 seem wasteful.

The style of presentation is best suited to a theoretician. To be sure, experimental and numerical results are presented with many photographs, figures and tables, but analysis and formulae are emphasized. Most of the problems are mathematical, few being exercises in physical reasoning. Also the author has in many sections, usually those closest to his own research interests, yielded to the temptation to add too much detail with copious references. Detail is often very useful for a specialist, but may confuse a student who is seeking a firm grasp of the essentials of the subject. This proliferation of detail is balanced, however, by the author's enthusiasm, which is particularly evident where his personal interests or acquaintance are relevant.

The changes in the present edition may have been limited by production costs. But textbooks on a subject as mature as fluid mechanics do not date rapidly, so there is no great loss on this account. However, consistent use of SI units is desirable now. Also, if a complete revision of the text were permissible, one would desire a more comprehensive updating of the topics with use of more recent work. In particular, the text might be better integrated with more numerical results, which have been produced so prolifically in the last decade or so. For example, the dated asymptotic analysis of the Orr–Sommerfeld equation on pages 485–494 could be advantageously replaced by an account of numerical results for stability characteristics of typical basic flows. On first acquaintance with hydrodynamic stability, a student needs to know the important concepts and results, with explicit values of the critical Reynolds numbers, rather than advanced theoretical methods.

In summary, then, the graduate student is well served by this comprehensive textbook on the theory of fluid mechanics, at a price low enough to induce him to buy a copy.

P. G. DRAZIN

# CORRIGENDUM

## Similarity of steady stratified flows

by CHIA-SHUN YIH

*J. Fluid Mech.* vol. 108, 1981, pp. 241–246

The line before equation (29) and equations (29) and (30) should read:

Let $D'/D't \equiv u'_\alpha \partial/\partial x_\alpha$ stand for the substantial derivative in the *associated flow*. Then, using (25) and

$$(D'/D't)\, dx_i = du'_i, \tag{29}$$

we have

$$\frac{D'}{D't}\Gamma' = \oint \left\{ \left( -\frac{1}{\rho_0}\frac{\partial p}{\partial x_i} - \frac{\rho}{\rho_0}\frac{\partial \Omega}{\partial x_i} \right) dx_i + d(\tfrac{1}{2}u'_\alpha u'_\alpha) \right\} = \oint \frac{\Omega}{\rho_0}\, d\rho. \tag{30}$$

Equation (44) should read:

$$D'\Gamma'/D't = \oint \Omega\, d\lambda.$$